Skew-symmetric tensor-spinor formulation of the spin 3/2 field

1973 J. Phys. A: Math. Nucl. Gen. 6 383

(http://iopscience.iop.org/0301-0015/6/3/012)

J. Phys. A: Math., Nucl. Gen., Vol. 6, March 1973. Printed in Great Britain. © 1973

Skew-symmetric tensor–spinor formulation of the spin $\frac{3}{2}$ field

Caroline Fisk† and W Tait‡

† Laboratoire de Physique Theorique, Institut Henri Poincaré, Paris, France
‡ Department of Physics, Queen Mary College, London E1 4NS, UK

MS received 1 November 1972

Abstract. The formulation of a field theory for a spin $\frac{3}{2}$ particle is sought in terms of a skew-symmetric tensor–spinor. If the wavefunction transforms irreducibly under the rotation group, the lagrangian density is not hermitian, but if it transforms as the direct sum of two spin $\frac{3}{2}$ components, this defect is removed. Furthermore, when minimal coupling to the electromagnetic field is introduced in the latter case, propagation remains causal, so that some improvement over the usual vector–spinor theory is achieved.

1. Introduction

The anomalies which appear in the field theory of higher spin particles in the presence of an interaction have been discussed by many authors (Johnson and Sudarshan 1961, Velo and Zwanziger 1969a, b, Schroer *et al* 1970, Minkowski and Seiler 1971, Shamaly and Capri 1972). Although these defects may be due to an inadequate formulation of the interaction Lagrangian, it is felt that the electromagnetic interaction is well enough understood to enable one to construct a suitable interaction term. But, in the case of the Rarita–Schwinger field for a spin $\frac{3}{2}$ particle minimally coupled to the electromagnetic field, it was shown by Johnson and Sudarshan (1961) that the equal time anticommutation relations between fields are not positive definite, and by Velo and Zwanziger (1969a) that the velocity of propagation of wavefronts is greater than the speed of light. Shamaly and Capri (1972) show that the problem is not resolved by the addition of (non-minimal) magnetic moment terms to the interaction Lagrangian.

In this article we derive alternative spin $\frac{3}{2}$ field equations in which the wavefunction is chosen to be an antisymmetric tensor–spinor. This particular choice of wavefunction is suggested by the fact that tensor–spinor equations may be obtained via the spin $\frac{3}{2}$ Bargmann–Wigner equations (Lurié 1968). To derive the Euler–Lagrange equations we use the method recently introduced by Aurilia and Umezawa (1967, 1969) which enables us to obtain, from a single wave equation, both the equation of motion and the necessary subsidiary conditions, including the antisymmetric property of the wavefunction.

In § 2 the spin projection operators which resolve the identity in the space of second rank tensor–spinor wavefunctions are constructed. The antisymmetric part is examined in § 3, and three formulations of spin $\frac{3}{2}$ are worked out using the projectors. In two of these formulations the wavefunctions transform irreducibly under the action of the Poincaré group, corresponding to a unique mass and spin, but the corresponding lagrangian densities are not hermitian. The third transforms reducibly as the direct sum of the two irreducible spin $\frac{3}{2}$ components having the same mass. In this case the lagrangian density is hermitian, and the subsidiary conditions serve to eliminate the two spin $\frac{1}{2}$ parts of the wavefunction.

Because of the defects in these theories (non-hermiticity or reducibility) they do not present a viable alternative to the Rarita–Schwinger theory for a free spin $\frac{3}{2}$ particle. However, the significance of our analysis appears in § 4 where we show for the case of the hermitian lagrangian density that no additional difficulties arise when minimal coupling to the electromagnetic field is introduced. It is demonstrated that the light cone is the only characteristic surface so that the requirements of causality are satisfied, that is, the speed of propagation is bounded by the speed of light. The calculation of the characteristic surface is carried out in a shock wave formalism (Stellmacher 1938, Madore and Tait 1973).

2. Projection operators

In this section we calculate the projection operators that project out of an arbitrary tensor–spinor $\psi^{\mu\nu}_{\gamma}$ parts which transform irreducibly under spatial rotations. Under spatial rotations an arbitrary tensor–spinor transforms according to the reducible representation

$$D(\tfrac{5}{2})\oplus 4D(\tfrac{3}{2})\oplus 5D(\tfrac{1}{2}).$$

For each of these 10 irreducible representations $D(s)$, there is a projection operator $P_i(s)$, $s = \frac{5}{2}, \frac{3}{2}, \frac{1}{2}$, which projects an arbitrary tensor–spinor into the space of the irreducible representation $D(s)$. To simplify the calculation of the projection operators we may resolve $\psi^{\mu\nu}_{\gamma}$ into a symmetric part and an antisymmetric part. The former transforms according to the representation

$$D(\tfrac{5}{2})\oplus 2D(\tfrac{3}{2})\oplus 3D(\tfrac{1}{2})$$

and the latter according to the representation

$$2D(\tfrac{3}{2})\oplus 2D(\tfrac{1}{2}).$$

If we designate by $S_i(s)$ and $A_i(s)$ the projection operators associated with the symmetric and antisymmetric subspaces respectively, we have

$$S^{\mu\nu}_{\sigma\rho} = S(\tfrac{5}{2})^{\mu\nu}_{\sigma\rho} + \sum_{i=1}^{2} S_i(\tfrac{3}{2})^{\mu\nu}_{\sigma\rho} + \sum_{i=1}^{3} S_i(\tfrac{1}{2})^{\mu\nu}_{\sigma\rho}$$

$$A^{\mu\nu}_{\sigma\rho} = \sum_{i=1}^{2} A_i(\tfrac{3}{2})^{\mu\nu}_{\sigma\rho} + \sum_{i=1}^{2} A_i(\tfrac{1}{2})^{\mu\nu}_{\sigma\rho}$$

where

$$S^{\mu\nu}_{\sigma\rho} = \tfrac{1}{2}(g^{\mu}_{\sigma}g^{\nu}_{\rho} + g^{\mu}_{\rho}g^{\nu}_{\sigma})$$

$$A^{\mu\nu}_{\sigma\rho} = \tfrac{1}{2}(g^{\mu}_{\sigma}g^{\nu}_{\rho} - g^{\mu}_{\rho}g^{\nu}_{\sigma}).$$

Following Aurilia and Umezawa, we construct the projection operators $S_i(s)$ and $A_i(s)$ by considering all possible products of the form

$$[D_i(s')\Gamma]P(s) \tag{2.1}$$

where the $D_i(s')$, $s' = 0, 1, 2$, are the spin projection operators in the tensor subspace which have been calculated elsewhere (Macfarlane and Tait 1972), and I is the identity operator in the spinor subspace. $P(s)$, $s = \frac{5}{2}, \frac{3}{2}, \frac{1}{2}$, projects the spin s part out of $\psi^{\mu\nu}_{\gamma}$ and is given by

$$P(s) = \prod_{s \neq s'} \frac{W - s'(s'+1)p^2}{s(s+1)p^2 - s'(s'+1)p^2} \tag{2.2}$$

where

$$p^2 = p_\mu p^\mu$$

and

$$\begin{aligned} W &= -w_\mu w^\mu \\ &= \tfrac{1}{2}p^2 S_{\alpha\beta}S^{\alpha\beta} - S_{\alpha\gamma}S^{\alpha\delta}p^\gamma p_\delta \end{aligned} \tag{2.3}$$

where w_μ is the Pauli–Lubanski pseudovector:

$$w_\mu = \tfrac{1}{2}\epsilon_{\mu\nu\sigma\rho}S^{\nu\sigma}p^\rho.$$

$S^{\alpha\beta}$ denotes the spin operator, that is,

$$(S^{\alpha\beta})^{\mu\nu}_{\sigma\rho} = (S_1^{\alpha\beta})^\mu_\sigma g^\nu_\rho + (S_1^{\alpha\beta})^\nu_\rho g^\mu_\sigma + \tfrac{1}{2}g^\mu_\sigma g^\nu_\rho \Sigma^{\alpha\beta} \tag{2.4}$$

where

$$(S_1^{\alpha\beta})^\mu_\sigma = \mathrm{i}(g^\alpha_\sigma g^{\beta\mu} - g^{\alpha\mu}g^\beta_\sigma) \tag{2.5}$$

and

$$\Sigma^{\alpha\beta} = -\tfrac{1}{2}\mathrm{i}(\gamma^\alpha\gamma^\beta - \gamma^\beta\gamma^\alpha) \tag{2.6}$$

(to simplify the notation we suppress the spinor index).

We are now in a position to calculate the projection operators. From (2.2) we find

$$P(\tfrac{5}{2}) = \frac{W^2 - \frac{9}{2}p^2W + \frac{45}{16}p^4}{40p^4} \tag{2.7}$$

$$P(\tfrac{3}{2}) = \frac{W^2 - \frac{19}{2}p^2W + \frac{105}{16}p^4}{-15p^4} \tag{2.8}$$

$$P(\tfrac{1}{2}) = \frac{W^2 - \frac{25}{2}p^2W + \frac{525}{16}p^4}{24p^4}. \tag{2.9}$$

Explicit expressions for $P(s)$ and hence for $A_i(s)$ using (2.1) are given in the Appendix. $S(\frac{5}{2}) = P(\frac{5}{2})$ is the only symmetric projector which is exhibited since it may prove useful for the evaluation of the $N^*_{5/2}$ contribution to πN scattering. The Feynman propagator is

$$\frac{\{(\gamma\cdot p + m)P(\frac{5}{2})\}_{p^2=m^2}}{p^2 - m^2}.$$

3. Three tensor–spinor formulations of spin $\frac{3}{2}$

3.1. Irreducible cases

By substituting the following expansion for a completely symmetric third rank spinor

$$\psi_{\alpha\beta\gamma} = (\gamma_\mu C)_{\alpha\beta}\psi^\mu_\gamma + \tfrac{1}{2}(\Sigma_{\mu\nu}C)_{\alpha\beta}\psi^{\mu\nu}_\gamma \tag{3.1}$$

into the Bargmann–Wigner equations, where C is the charge conjugation matrix, ψ^{μ}_{γ} is a vector–spinor and $\psi^{\mu\nu}_{\gamma}$ a skew-symmetric tensor–spinor, we can by expressing ψ^{μ} in terms of $\psi^{\alpha\beta}$, derive the following equations for $\psi^{\mu\nu}$:

$$\begin{aligned}
&(\gamma \cdot p - m)\psi^{\mu\nu} = 0\\
&\gamma_{\mu}\gamma_{\nu}\psi^{\mu\nu} = 0\\
&\epsilon^{\mu\nu}{}_{\sigma\rho}p_{\nu}\psi^{\sigma\rho} = 0.
\end{aligned} \tag{3.2}$$

A different expansion of the spinor

$$\psi_{\alpha\beta\gamma} = (\gamma_{\mu}C)_{\alpha\beta}\psi^{\mu}_{\gamma} + \tfrac{1}{2}(\gamma^{5}\Sigma_{\mu\nu}C)_{\alpha\beta}\chi^{\mu\nu}_{\gamma} \tag{3.3}$$

is valid since $\gamma^{5}\Sigma_{\mu\nu}C$ is also symmetric, and because of the relation

$$\gamma^{5}\Sigma_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\sigma\rho}\Sigma^{\sigma\rho}$$

we have that χ is the dual of ψ. In this case the relevant equations are

$$\begin{aligned}
&(\gamma \cdot p - m)\chi^{\mu\nu} = 0\\
&p_{\mu}\chi^{\mu\nu} = 0\\
&\gamma_{\mu}\gamma_{\nu}\chi^{\mu\nu} = 0.
\end{aligned} \tag{3.4}$$

It is easily checked that (3.2) and (3.4) considered in the rest frame lead to the correct number of degrees of freedom for a spin $\frac{3}{2}$ particle.

Using the method of Aurilia and Umezawa (1967, 1969) we proceed to find a single lagrangian equation which will imply (3.4) (or (3.2)). The kernel $\Lambda(p)$ of the lagrangian

$$\mathscr{L} = \bar{\chi}^{\mu\nu}\Lambda^{\alpha\beta}_{\mu\nu}\chi_{\alpha\beta}$$

is given by

$$\Lambda(p) = -(\gamma \cdot p - m)A_{1}(\tfrac{3}{2}) - m(a_{1}S + a_{2}A_{2}(\tfrac{3}{2}) + a_{3}A_{1}(\tfrac{1}{2}) + a_{4}A_{2}(\tfrac{1}{2})) \tag{3.5}$$

and the Klein–Gordan divisor by

$$d(p) = (\gamma \cdot p + m)A_{1}(\tfrac{3}{2}) + \frac{p^{2} - m^{2}}{m}(a_{1}^{-1}S + a_{2}^{-1}A_{2}(\tfrac{3}{2}) + a_{3}^{-1}A_{1}(\tfrac{1}{2}) + a_{4}^{-1}A_{2}(\tfrac{1}{2})) \tag{3.6}$$

where a_{i}, $i = 1, 2, 3, 4$, are non-vanishing constants. By choosing $a_{2} = a_{3} = a_{4} = -1$, we cancel all the singular terms (ie inverse powers of p^{2}) in (3.5) except those of the form

$$\frac{\gamma \cdot p}{p^{2}}p^{\mu}p_{\sigma}\gamma^{\nu}\gamma_{\rho} \qquad \text{and} \qquad \frac{\gamma \cdot p}{p^{2}}p^{\mu}p_{\sigma}g^{\nu}_{\rho}.$$

These terms may be removed by multiplying the kernel on the left by a non-singular matrix η which is made up of products of the form

$$I + A_{i}(s)MA_{j}(s'). \tag{3.7}$$

The matrix M may contain any possible combination of γ^{μ}, $g^{\mu\nu}$, $\epsilon^{\mu\nu}{}_{\sigma\rho}$, p_{μ}, and η is determined such that the singular terms generated by η cancel those contained in Λ. We find that all terms of the form

$$A_{i}(s)MA_{j}(s')$$

vanish if $s \neq s'$. In addition, all non-vanishing terms of the form $A_{i}(s)MA_{j}(s)$ that generate the singular terms contained in Λ, are proportional to

$$A_i(s)^{\mu\nu}_{\alpha\beta}\gamma^\alpha g^\beta_\gamma \gamma_\delta A_j(s)^{\gamma\delta}_{\sigma\rho}.$$

If we define

$$\begin{aligned}
X^{\mu\nu}_{\sigma\rho} &= A_2(\tfrac{1}{2})^{\mu\nu}_{\alpha\beta}\gamma^\beta\gamma_\delta A_1(\tfrac{1}{2})^{\alpha\delta}_{\sigma\rho}\\
Y^{\mu\nu}_{\sigma\rho} &= A_1(\tfrac{1}{2})^{\mu\nu}_{\alpha\beta}\gamma^\beta\gamma_\delta A_2(\tfrac{1}{2})^{\alpha\delta}_{\sigma\rho}\\
Z^{\mu\nu}_{\sigma\rho} &= A_1(\tfrac{3}{2})^{\mu\nu}_{\alpha\beta}\gamma^\beta\gamma_\delta A_2(\tfrac{3}{2})^{\alpha\delta}_{\sigma\rho}\\
Z'^{\mu\nu}_{\sigma\rho} &= A_2(\tfrac{3}{2})^{\mu\nu}_{\alpha\beta}\gamma^\beta\gamma_\delta A_1(\tfrac{3}{2})^{\alpha\delta}_{\sigma\rho}.
\end{aligned}$$

(3.7) contains products of the forms

$$I+n_1X, \qquad I+n_2Y, \qquad I+n_3Z, \qquad I+n_4Z'$$

where $n_i = c_i + d_i(\gamma\,.\,p/m)$. We should also like to choose η so that the lagrangian

$$\mathscr{L} = \bar{\psi}\eta\Lambda\psi$$

may be made hermitian. However, we find that it is not possible to eliminate the singular terms from the wave equation and, at the same time, obtain a hermitian lagrangian for any choice, real or complex, of the parameters n_i. The singular terms will cancel if we multiply Λ by the matrix

$$\eta = \left(I + c_1\frac{\gamma\,.\,p}{m}Y\right)\left(I + c_2\frac{\gamma\,.\,p}{m}Z\right)$$

and put $a_2 = a_3 = a_4 = c_1 = -1$, $c_2 = -2$. If $\tilde{\Lambda} = \eta\Lambda$, the wave equation is then $\tilde{\Lambda}\chi = 0$, that is,

$$\begin{aligned}
\{ &-\tfrac{2}{3}\gamma\,.\,p(g^\mu_\sigma g^\nu_\rho - g^\mu_\rho g^\nu_\sigma) - \tfrac{1}{6}\gamma\,.\,p(\gamma^\mu\gamma_\rho g^\nu_\sigma - \gamma^\nu\gamma_\rho g^\mu_\sigma - \gamma^\mu\gamma_\sigma g^\nu_\rho + \gamma^\nu\gamma_\sigma g^\mu_\rho) - \tfrac{1}{6}(\gamma^\mu\gamma^\nu - g^{\mu\nu})(p_\sigma\gamma_\rho - p_\rho\gamma_\sigma)\\
&- \tfrac{2}{3}(g^\mu_\rho\gamma^\nu p_\sigma - g^\nu_\rho\gamma^\mu p_\sigma - g^\mu_\sigma\gamma^\nu p_\rho + g^\nu_\sigma\gamma^\mu p_\rho) + \tfrac{1}{6}(g^\mu_\rho p^\nu\gamma_\sigma - g^\nu_\rho p^\mu\gamma_\sigma - g^\mu_\sigma p^\nu\gamma_\rho + g^\nu_\sigma p^\mu\gamma_\rho)\\
&+ \tfrac{1}{2}m(g^\mu_\sigma g^\nu_\rho - g^\mu_\rho g^\nu_\sigma) - \tfrac{1}{2}a_1 m(g^\mu_\sigma g^\nu_\rho + g^\mu_\rho g^\nu_\sigma)\}\chi^{\sigma\rho} = 0.
\end{aligned} \tag{3.8}$$

The Klein–Gordan divisor is given by $\tilde{d} = d\eta^{-1}$, that is,

$$\begin{aligned}
\tilde{d}^{\mu\nu}_{\sigma\rho} = (\gamma\,.\,p + m)\Bigg(&\tfrac{2}{3}(g^\mu_\sigma g^\nu_\rho - g^\mu_\rho g^\nu_\sigma) + \tfrac{1}{6}(g^\nu_\sigma\gamma^\mu\gamma_\rho + g^\mu_\rho\gamma^\nu\gamma_\sigma - g^\mu_\sigma\gamma^\nu\gamma_\rho - g^\nu_\rho\gamma^\mu\gamma_\sigma)\\
&+ \frac{1}{6m}(g^\nu_\rho\gamma_\sigma p^\mu + g^\mu_\sigma\gamma_\rho p^\nu - g^\mu_\rho\gamma_\sigma p^\nu - g^\nu_\sigma\gamma_\rho p^\mu)\\
&+ \frac{2}{3m}(g^\mu_\rho\gamma^\nu p_\sigma - g^\mu_\sigma\gamma^\nu p_\rho - g^\nu_\rho\gamma^\mu p_\sigma + g^\nu_\sigma\gamma^\mu p_\rho) + \frac{1}{3m}(\gamma^\mu\gamma^\nu - g^{\mu\nu})(p_\sigma\gamma_\rho - p_\rho\gamma_\sigma)\\
&- \frac{1}{6m^2}(p^\mu p_\sigma\gamma^\nu\gamma_\rho + p^\nu p_\rho\gamma^\mu\gamma_\sigma - p^\nu p_\sigma\gamma^\mu\gamma_\rho - p^\mu p_\rho\gamma^\nu\gamma_\sigma)\Bigg)
\end{aligned}$$

$$+\frac{p^2-m^2}{m}\left(\frac{1}{2a_1}(g^\mu_\sigma g^\nu_\rho+g^\mu_\rho g^\nu_\sigma)+\tfrac{1}{6}(g^\mu_\sigma g^\nu_\rho-g^\mu_\rho g^\nu_\sigma)\right.$$

$$\left.+\tfrac{1}{6}(g^\nu_\sigma\gamma^\mu\gamma_\rho+g^\mu_\rho\gamma^\nu\gamma_\sigma-g^\mu_\sigma\gamma^\nu\gamma_\rho-g^\nu_\rho\gamma^\mu\gamma_\sigma)+\frac{1}{6m}(\gamma^\mu\gamma^\nu-g^{\mu\nu})(p_\sigma\gamma_\rho-p_\rho\gamma_\sigma)\right). \quad (3.9)$$

From (3.9) we see that the Green function on the mass shell may be written

$$G=\frac{\{(\gamma\cdot p+m)(A_1(\tfrac{3}{2})+2Z)\}_{p^2=m^2}}{p^2-m^2} \quad (3.10)$$

which is not equal to the Feynman propagator which is the expression (3.10) without the term $2Z$.

In a similar manner, we may obtain a second wave equation by exchanging $A_1(\tfrac{3}{2})$ and $A_2(\tfrac{3}{2})$ in (3.5). In this case we have

$$\Lambda(p)=-(\gamma\cdot p-m)A_2(\tfrac{3}{2})-m(b_1S+b_2A_1(\tfrac{3}{2})+b_3A_1(\tfrac{1}{2})+b_4A_2(\tfrac{1}{2})).$$

Using the same argument as that used in the previous case, we find that we may remove the singular terms by multiplying Λ by the matrix

$$\eta=\left(I+d_1\frac{\gamma\cdot p}{m}X\right)\left(I+d_2\frac{\gamma\cdot p}{m}Z'\right)$$

but that it is not possible to construct a hermitian lagrangian. The singular terms in $\tilde{\Lambda}=\eta\Lambda$ cancel if we take $b_2=b_3=b_4=d_1=-1$, $d_2=-2$. The wave equation becomes $\tilde{\Lambda}\psi=0$, that is,

$$\{\tfrac{1}{2}m(g^\mu_\sigma g^\nu_\rho-g^\mu_\rho g^\nu_\sigma)-\tfrac{1}{2}mb_1(g^\mu_\sigma g^\nu_\rho+g^\mu_\rho g^\nu_\sigma)-\tfrac{1}{6}(p^\mu\gamma^\nu-p^\nu\gamma^\mu)(\gamma_\sigma\gamma_\rho-g_{\sigma\rho})$$

$$-\tfrac{1}{2}(g^\nu_\rho\gamma_\sigma p^\mu-g^\mu_\rho\gamma_\sigma p^\nu+g^\mu_\sigma\gamma_\rho p^\nu-g^\nu_\sigma\gamma_\rho p^\mu)\}\psi^{\sigma\rho}=0. \quad (3.11)$$

The Klein–Gordon divisor is

$$\tilde{d}^{\mu\nu}_{\sigma\rho}=(\gamma\cdot p+m)\left(\frac{1}{2m}(g^\nu_\rho\gamma_\sigma p^\mu-g^\mu_\rho\gamma_\sigma p^\nu+g^\mu_\sigma\gamma_\rho p^\nu-g^\nu_\sigma\gamma_\rho p^\mu)+\frac{1}{3m}(p^\mu\gamma^\nu-p^\nu\gamma^\mu)(\gamma_\sigma\gamma_\rho-g_{\sigma\rho})\right.$$

$$\left.+\frac{1}{6m^2}(p^\mu p_\sigma\gamma^\nu\gamma_\rho+p^\nu p_\rho\gamma^\mu\gamma_\sigma-p^\nu p_\sigma\gamma^\mu\gamma_\rho-p^\mu p_\rho\gamma^\nu\gamma_\sigma)\right)$$

$$+\frac{p^2-m^2}{m}\left(\frac{1}{2b_1}(g^\mu_\sigma g^\nu_\rho+g^\mu_\rho g^\nu_\sigma)-\tfrac{1}{2}(g^\mu_\sigma g^\nu_\rho-g^\mu_\rho g^\nu_\sigma)\right.$$

$$\left.+\frac{1}{6m}(p^\mu\gamma^\nu-p^\nu\gamma^\mu)(\gamma_\sigma\gamma_\rho-g_{\sigma\rho})\right) \quad (3.12)$$

and in this case the Green function on the mass shell is

$$G=\frac{\{(\gamma\cdot p+m)(A_2(\tfrac{3}{2})+2Z')\}_{p^2=m^2}}{p^2-m^2}. \quad (3.13)$$

3.2. Reducible case

We conclude this section by noting that we can derive a third equation in which the wavefunction is reducible, being the direct sum of the two irreducible components.

Putting $A(\frac{3}{2}) = \Sigma_{i=1}^{2} A_i(\frac{3}{2})$, the kernel is

$$\Lambda(p) = -(\gamma \cdot p - m)A(\tfrac{3}{2}) - m(g_1 S + g_2 A_1(\tfrac{1}{2}) + g_3 A_2(\tfrac{1}{2})).$$

The singular terms cancel when $g_2 = g_3 = -1$, and the wave equation is

$$\begin{aligned}\{ &-\tfrac{2}{3}\gamma \cdot p(g_\sigma^\mu g_\rho^\nu - g_\rho^\mu g_\sigma^\nu) - \tfrac{1}{6}\gamma \cdot p(g_\sigma^\nu \gamma^\mu \gamma_\rho + g_\rho^\mu \gamma^\nu \gamma_\sigma - g_\sigma^\mu \gamma^\nu \gamma_\rho - g_\rho^\nu \gamma^\mu \gamma_\sigma) \\ &+ \tfrac{1}{6}(g_\rho^\nu \gamma^\mu p_\sigma - g_\rho^\nu \gamma_\sigma p^\mu + g_\sigma^\mu \gamma^\nu p_\rho - g_\sigma^\mu \gamma_\rho p^\nu \\ &- g_\rho^\mu \gamma^\nu p_\sigma + g_\rho^\mu \gamma_\sigma p^\nu - g_\sigma^\nu \gamma^\mu p_\rho + g_\sigma^\nu \gamma_\rho p^\mu) \\ &+ \tfrac{1}{2}m(g_\sigma^\mu g_\rho^\nu - g_\sigma^\mu g_\sigma^\nu) - m g_1(g_\sigma^\mu g_\rho^\nu + g_\rho^\mu g_\sigma^\nu)\}\phi^{\sigma\rho} = 0. \end{aligned} \tag{3.14}$$

If we multiply respectively by $S_{\mu\nu}^{\alpha\beta}$, $\gamma_\mu \gamma_\nu$, $\gamma_\mu p_\nu$ we obtain the subsidiary conditions

$$\begin{aligned} &\phi^{\mu\nu} = -\phi^{\nu\mu} \\ &\gamma_\sigma \gamma_\rho \phi^{\sigma\rho} = 0 \\ &p_\sigma \gamma_\sigma \phi^{\sigma\rho} = 0 \end{aligned}$$

and in this case (3.14) may be derived from a *hermitian* lagrangian density. The Klein–Gordan divisor is then

$$\begin{aligned} d = (\gamma \cdot p + m)\Big(&\tfrac{2}{3}(g_\sigma^\mu g_\rho^\nu - g_\rho^\mu g_\sigma^\nu) + \tfrac{1}{6}(g_\sigma^\nu \gamma^\mu \gamma_\rho - g_\sigma^\mu \gamma^\nu \gamma_\rho - g_\rho^\nu \gamma^\mu \gamma_\sigma + g_\rho^\mu \gamma^\nu \gamma_\sigma) \\ &+ \frac{1}{6m}(g_\rho^\nu \gamma_\sigma p^\mu - g_\rho^\mu \gamma_\sigma p^\nu - g_\sigma^\nu \gamma_\rho p^\mu + g_\sigma^\mu \gamma_\rho p^\nu - g_\rho^\nu \gamma^\mu p_\sigma + g_\rho^\mu \gamma^\nu p_\sigma + g_\sigma^\nu \gamma^\mu p_\rho - g_\sigma^\mu \gamma^\nu p_\rho)\Big) \\ &+ \frac{p^2 - m^2}{m}\Big(\tfrac{1}{6}(g_\sigma^\mu g_\rho^\nu - g_\rho^\mu g_\sigma^\nu) + \tfrac{1}{6}(g_\sigma^\nu \gamma^\mu \gamma_\rho - g_\sigma^\mu \gamma^\nu \gamma_\rho - g_\rho^\nu \gamma^\mu \gamma_\sigma + g_\rho^\mu \gamma^\nu \gamma_\sigma) \\ &- \frac{1}{2g_1}(g_\sigma^\mu g_\rho^\nu + g_\rho^\mu g_\sigma^\nu)\Big). \end{aligned}$$

4. Minimal electromagnetic interaction

With the minimal coupling of the spin $\frac{3}{2}$ field to the electromagnetic field effected by the substitution $p_\mu \to \pi_\mu = p_\mu + eA_\mu$ we must determine whether or not the wave propagation is causal. This is done by calculating the characteristic surface of the coupled wave equation, to see if it lies inside or outside the light cone (Velo and Zwanziger 1969a). We perform this calculation in a shock-wave formalism which was applied by one of the authors (Madore and Tait 1973) to other interacting higher spin systems.

For a first order wave equation, the characteristic surface is one which will support a discontinuity in the first derivative of the wavefunction. Let σ be a smooth hypersurface given in a region of space–time, where by smooth we mean differentiable of class $\mathscr{C}^n, n \geqslant 3$; and let $z(x^\mu)$ be a real valued smooth function of x^μ regular in a neighbourhood U of σ, and vanishing on σ. The hypersurface σ divides U into two regions U^+ and U^- corresponding to $z \geqslant 0$ and $z < 0$ respectively. Define $\xi_\mu = \partial_\mu z$. ξ_μ is non-vanishing in U and normal to σ.

Consider a wavefunction $\phi(x^\mu)$ defined in U and smooth in the interior of U^+ and U^-. We suppose that ϕ is continuous, but has a discontinuity in the first and possibly higher derivatives across σ. Let ϕ be denoted by $\phi^{\pm}$ in the regions $U^{\pm}$. By extending $\phi^{\pm}$ smoothly into $U^{\mp}$, the discontinuity $[\phi] = \phi^{+} - \phi^{-}$ may be defined as a smooth function in U. There exist uncountably many smooth functions $\tilde{k}, \tilde{f}$ defined in U such that

$$[\phi] = z\tilde{k} + \tfrac{1}{2}z^{2}\tilde{f}. \tag{4.1}$$

For a function $\tilde{k}$ defined in U, let $\tilde{k}_{|\sigma}$ be its restriction to σ. Define k, f by

$$k = \tilde{k}_{|\sigma}, \qquad f = \tilde{f}_{|\sigma}.$$

From (4.1) we can calculate the discontinuities across σ in the first and higher derivatives of ϕ in an arbitrary direction, for example,

$$\begin{aligned}
[\partial_{\alpha}\phi] &\equiv (\partial_{\alpha}[\phi])_{|\sigma} = \xi_{\alpha}k \\
[\partial_{\beta}\partial_{\alpha}\phi] &\equiv (\partial_{\beta}\partial_{\alpha}[\phi])_{|\sigma} = \xi_{\beta}\xi_{\alpha}f + \xi_{(\alpha}\partial_{\beta)}\tilde{k} + \partial_{\beta}\xi_{\alpha}k.
\end{aligned} \tag{4.2}$$

Using (4.2) we investigate equation (3.14) for the reducible wavefunction derived from a hermitian lagrangian density. After the antisymmetry property has been derived, this equation reduces to

$$\begin{aligned}
R^{\mu\nu} = {} & -\tfrac{4}{3}\gamma\cdot\pi\phi^{\mu\nu} - \tfrac{1}{3}\,\gamma\cdot\pi(\gamma^{\mu}\gamma_{\rho}g^{\nu}_{\sigma} - \gamma^{\nu}\gamma_{\rho}g^{\mu}_{\sigma})\phi^{\sigma\rho} \\
& + \tfrac{1}{3}(\gamma^{\mu}\pi_{\sigma}g^{\nu}_{\rho} - \gamma^{\nu}\pi_{\sigma}g^{\mu}_{\rho} - \pi^{\mu}\gamma_{\sigma}g^{\nu}_{\rho} + \pi^{\nu}\gamma_{\sigma}g^{\mu}_{\rho})\phi^{\sigma\rho} + m\phi^{\mu\nu} = 0.
\end{aligned} \tag{4.3}$$

Contracting with $\gamma_{\mu}\gamma_{\nu}$, $\pi_{\mu}\gamma_{\nu}$ we obtain the following:

$$\begin{aligned}
d &= \gamma_{\mu}\gamma_{\nu}\phi^{\mu\nu} = 0 \\
d' &= m\pi_{\mu}\gamma_{\nu}\phi^{\mu\nu} - eF_{\mu\rho}\Sigma^{\rho}{}_{\nu}\phi^{\mu\nu} = 0
\end{aligned}$$

and from the expressions $[R^{\mu\nu}]$, $[\partial_{\alpha}d]$, $[d']$, we get

$$-\tfrac{4}{3}\gamma\cdot\xi k^{\mu\nu} - \tfrac{1}{3}\gamma\cdot\xi(\gamma^{\mu}\gamma_{\rho}g^{\nu}_{\sigma} - \gamma^{\nu}\gamma_{\rho}g^{\mu}_{\sigma})k^{\sigma\rho} + \tfrac{1}{3}(\gamma^{\mu}\xi_{\sigma}g^{\nu}_{\rho} - \gamma^{\nu}\xi_{\sigma}g^{\mu}_{\rho} - \xi^{\mu}\gamma_{\sigma}g^{\nu}_{\rho} + \xi^{\nu}\gamma_{\sigma}g^{\mu}_{\rho})k^{\sigma\rho} = 0 \tag{4.4}$$

$$\gamma_{\mu}\gamma_{\nu}k^{\mu\nu} = 0 \tag{4.5}$$

$$\xi_{\mu}\gamma_{\nu}k^{\mu\nu} = 0. \tag{4.6}$$

Contracting (4.4) with γ_{μ} and ξ_{μ} and using (4.5), (4.6) yields, assuming $\xi^{2} \neq 0$,

$$\xi_{\mu}k^{\mu\nu} = \gamma_{\mu}k^{\mu\nu} = 0. \tag{4.7}$$

(4.7) in (4.4) gives immediately that $k^{\mu\nu} = 0$. This means that a discontinuity $k^{\mu\nu}$ can only exist across the light cone, that is, (4.3) has light cone characteristics so that the maximum propagation velocity is the speed of light. This result is easily understood when we realize that the constraint d is unaffected by the interaction, while d' contains no derivative terms other than the one which appears in the free field case. Since there are no derivative terms in the interaction, it follows that the characteristic surface remains the light cone, where the discontinuity obeys the equation

$$\gamma\cdot\xi\xi^{[\nu}\gamma_{\sigma}k^{\sigma\mu]} = 0.$$

Acknowledgments

Part of this work was carried out during the tenure of an Exchange Fellowship, sponsored by the Royal Society and the Centre National de la Recherche Scientifique, by one of the authors (WT).

Appendix

Using equation (2.3), the Casimir operator W^2 is calculated to be

$$
\begin{aligned}
W^{2\mu\nu}_{\sigma\rho} = {} & p^4(\tfrac{761}{16}g^\mu_\sigma g^\nu_\rho - 17g^{\mu\nu}g_{\sigma\rho} + 29g^\nu_\sigma g^\mu_\rho - \tfrac{17}{2}g^\mu_\sigma\gamma^\nu\gamma_\rho - \tfrac{17}{2}g^\nu_\rho\gamma^\mu\gamma_\sigma - 4g^\nu_\sigma\gamma^\mu\gamma_\rho - 4g^\mu_\rho\gamma^\nu\gamma_\sigma) \\
& + p^2(17g^{\mu\nu}p_\sigma p_\rho + 17g_{\sigma\rho}p^\mu p^\nu - 25g^\nu_\rho p^\mu p_\sigma - 25g^\mu_\sigma p^\nu p_\rho - 25g^\mu_\rho p^\nu p_\sigma - 25g^\nu_\sigma p^\mu p_\rho) \\
& + p^2(\tfrac{17}{2}g^\nu_\rho\gamma^\mu p_\sigma - \tfrac{17}{2}g^\nu_\rho\gamma_\sigma p^\mu + \tfrac{17}{2}g^\mu_\sigma\gamma^\nu p_\rho - \tfrac{17}{2}g^\mu_\sigma p^\nu\gamma_\rho \\
& + 4g^\mu_\rho\gamma^\nu p_\sigma - 4g^\mu_\rho\gamma_\sigma p^\nu + 4g^\nu_\sigma\gamma^\mu p_\rho - 4g^\nu_\sigma\gamma_\rho p^\mu)\gamma\cdot p \\
& + 4p^2(\gamma^\nu\gamma_\rho p^\mu p_\sigma + \gamma^\nu\gamma_\sigma p^\mu p_\rho + \gamma^\mu\gamma_\rho p^\nu p_\sigma + \gamma^\mu\gamma_\sigma p^\nu p_\rho) \\
& + 8(p^\mu p^\nu p_\sigma\gamma_\rho + p^\mu p^\nu p_\rho\gamma_\sigma - p^\mu p_\sigma p_\rho\gamma^\nu - p^\nu p_\sigma p_\rho\gamma^\mu)\gamma\cdot p + 16p^\mu p^\nu p_\sigma p_\rho.
\end{aligned}
$$

From equations (2.7), (2.8), (2.9), (2.1), the following projection operators have been calculated: $P(\frac{5}{2})$, $P(\frac{3}{2})$, $P(\frac{1}{2})$; $A_1(\frac{3}{2})$, $A_2(\frac{3}{2})$, $A_1(\frac{1}{2})$, $A_2(\frac{1}{2})$. We have

$$
\begin{aligned}
P(\tfrac{5}{2})^{\mu\nu}_{\sigma\rho} = {} & \tfrac{1}{2}(g^\mu_\sigma g^\nu_\rho + g^\mu_\rho g^\nu_\sigma) - \tfrac{1}{5}g^{\mu\nu}g_{\sigma\rho} - \tfrac{1}{10}(g^\mu_\sigma\gamma^\nu\gamma_\rho + g^\nu_\rho\gamma^\mu\gamma_\sigma + g^\nu_\sigma\gamma^\mu\gamma_\rho + g^\mu_\rho\gamma^\nu\gamma_\sigma) \\
& + \frac{1}{5p^2}(g^{\mu\nu}p_\sigma p_\rho + g_{\sigma\rho}p^\mu p^\nu) - \frac{2}{5p^2}(g^\nu_\rho p^\mu p_\sigma + g^\mu_\sigma p^\nu p_\rho + g^\mu_\rho p^\nu p_\sigma + g^\nu_\sigma p^\mu p_\rho) \\
& + \frac{1}{10p^2}(g^\nu_\rho\gamma^\mu p_\sigma - g^\nu_\rho\gamma_\sigma p^\mu + g^\mu_\sigma\gamma^\nu p_\rho \\
& - g^\mu_\sigma\gamma_\rho p^\nu + g^\mu_\rho\gamma^\nu p_\sigma - g^\mu_\rho\gamma_\sigma p^\nu + g^\nu_\sigma\gamma^\mu p_\rho - g^\nu_\sigma\gamma_\rho p^\mu)\gamma\cdot p \\
& + \frac{1}{10p^2}(\gamma^\nu\gamma_\rho p^\mu p_\sigma + \gamma^\nu\gamma_\sigma p^\mu p_\rho + \gamma^\mu\gamma_\sigma p^\nu p_\rho + \gamma^\mu\gamma_\rho p^\nu p_\sigma) \\
& + \frac{1}{5p^4}(\gamma_\rho p^\mu p^\nu p_\sigma + \gamma_\sigma p^\mu p^\nu p_\rho - \gamma^\nu p^\mu p_\sigma p_\rho - \gamma^\mu p^\nu p_\sigma p_\rho)\gamma\cdot p + \frac{2}{5p^4}p^\mu p^\nu p_\sigma p_\rho
\end{aligned}
$$

$$
\begin{aligned}
P(\tfrac{3}{2})^{\mu\nu}_{\sigma\rho} = {} & \tfrac{2}{3}(g^\mu_\sigma g^\nu_\rho - g^\nu_\sigma g^\mu_\rho) - \tfrac{2}{15}g^{\mu\nu}g_{\sigma\rho} - \tfrac{1}{15}(g^\mu_\sigma\gamma^\nu\gamma_\rho + g^\nu_\rho\gamma^\mu\gamma_\sigma) + \tfrac{4}{15}(g^\nu_\sigma\gamma^\mu\gamma_\rho + g^\mu_\rho\gamma^\nu\gamma_\sigma) \\
& + \frac{2}{15p^2}(g^{\mu\nu}p_\sigma p_\rho + g_{\sigma\rho}p^\mu p^\nu) + \frac{2}{5p^2}(g^\nu_\rho p^\mu p_\sigma + g^\mu_\sigma p^\nu p_\rho + g^\mu_\rho p^\nu p_\sigma + g^\nu_\sigma p^\mu p_\rho) \\
& + \frac{1}{15p^2}(g^\nu_\rho\gamma^\mu p_\sigma - g^\nu_\rho\gamma_\sigma p^\mu + g^\mu_\sigma\gamma^\nu p_\rho - g^\mu_\sigma\gamma_\rho p^\nu)\gamma\cdot p \\
& - \frac{4}{15p^2}(g^\mu_\rho\gamma^\nu p_\sigma - g^\mu_\rho\gamma_\sigma p^\nu + g^\nu_\sigma\gamma^\mu p_\rho - g^\nu_\sigma\gamma_\rho p^\mu)\gamma\cdot p \\
& - \frac{4}{15p^2}(\gamma^\nu\gamma_\rho p^\mu p_\sigma + \gamma^\nu\gamma_\sigma p^\mu p_\rho + \gamma^\mu\gamma_\sigma p^\nu p_\rho + \gamma^\mu\gamma_\rho p^\nu p_\sigma) \\
& - \frac{8}{15p^4}(\gamma_\rho p^\mu p^\nu p_\sigma + \gamma_\sigma p^\mu p^\nu p_\rho - \gamma^\nu p^\mu p_\sigma p_\rho - \gamma^\mu p^\nu p_\sigma p_\rho)\gamma\cdot p - \frac{16}{15p^4}(p^\mu p^\nu p_\sigma p_\rho)
\end{aligned}
$$

$$
\begin{aligned}
P(\tfrac{1}{2})^{\mu\nu}_{\sigma\rho} = {} & -\tfrac{1}{6}(g^{\mu}_{\sigma}g^{\nu}_{\rho}-g^{\nu}_{\sigma}g^{\mu}_{\rho})+\tfrac{1}{3}g^{\mu\nu}g_{\sigma\rho}+\tfrac{1}{6}(g^{\mu}_{\sigma}\gamma^{\nu}\gamma_{\rho}+g^{\nu}_{\rho}\gamma^{\mu}\gamma_{\sigma}-g^{\nu}_{\sigma}\gamma^{\mu}\gamma_{\rho}-g^{\mu}_{\rho}\gamma^{\nu}\gamma_{\sigma}) \\
& -\frac{1}{3p^2}(g^{\mu\nu}p_{\sigma}p_{\rho}+g_{\sigma\rho}p^{\mu}p^{\nu}) \\
& -\frac{1}{6p^2}(g^{\nu}_{\rho}\gamma^{\mu}p_{\sigma}-g^{\nu}_{\rho}\gamma_{\sigma}p^{\mu}+g^{\mu}_{\sigma}\gamma^{\nu}p_{\rho}-g^{\mu}_{\sigma}\gamma_{\rho}p^{\nu}-g^{\mu}_{\rho}\gamma^{\nu}p_{\sigma}+g^{\mu}_{\rho}\gamma_{\sigma}p^{\nu}-g^{\nu}_{\sigma}\gamma^{\mu}p_{\rho} \\
& +g^{\nu}_{\sigma}\gamma_{\rho}p^{\mu})\gamma\cdot p+\frac{1}{6p^2}(\gamma^{\nu}\gamma_{\rho}p^{\mu}p_{\sigma}+\gamma^{\nu}\gamma_{\sigma}p^{\mu}p_{\rho}+\gamma^{\mu}\gamma_{\sigma}p^{\nu}p_{\rho}+\gamma^{\mu}\gamma_{\rho}p^{\nu}p_{\sigma}) \\
& +\frac{1}{3p^4}(\gamma_{\rho}p^{\mu}p^{\nu}p_{\sigma}+\gamma_{\sigma}p^{\mu}p^{\nu}p_{\rho}-\gamma^{\nu}p^{\mu}p_{\sigma}p_{\rho}-\gamma^{\mu}p^{\nu}p_{\sigma}p_{\rho})\gamma\cdot p+\frac{2}{3p^4}p^{\mu}p^{\nu}p_{\sigma}p_{\rho}
\end{aligned}
$$

$$
\begin{aligned}
A_1(\tfrac{3}{2})^{\mu\nu}_{\sigma\rho} = {} & \tfrac{2}{3}(g^{\mu}_{\sigma}g^{\nu}_{\rho}-g^{\mu}_{\rho}g^{\nu}_{\sigma})+\tfrac{1}{6}(g^{\nu}_{\sigma}\gamma^{\mu}\gamma_{\rho}+g^{\mu}_{\rho}\gamma^{\nu}\gamma_{\sigma}-g^{\mu}_{\sigma}\gamma^{\nu}\gamma_{\rho}-g^{\nu}_{\rho}\gamma^{\mu}\gamma_{\sigma}) \\
& +\frac{1}{2p^2}(g^{\mu}_{\rho}p^{\nu}p_{\sigma}+g^{\nu}_{\sigma}p^{\mu}p_{\rho}-p^{\mu}p_{\sigma}g^{\nu}_{\rho}-g^{\mu}_{\sigma}p^{\nu}p_{\rho}) \\
& +\frac{1}{6p^2}(\gamma^{\nu}\gamma_{\rho}p^{\mu}p_{\sigma}+\gamma^{\mu}\gamma_{\sigma}p^{\nu}p_{\rho}-\gamma^{\mu}\gamma_{\rho}p^{\nu}p_{\sigma}-\gamma^{\nu}\gamma_{\sigma}p^{\mu}p_{\rho}) \\
& +\frac{\gamma\cdot p}{6p^2}(g^{\nu}_{\rho}\gamma_{\sigma}p^{\mu}-g^{\nu}_{\rho}\gamma^{\mu}p_{\sigma}+g^{\mu}_{\sigma}\gamma_{\rho}p^{\nu}-g^{\mu}_{\sigma}\gamma^{\nu}p_{\rho}+g^{\mu}_{\rho}\gamma^{\nu}p_{\sigma}-g^{\mu}_{\rho}\gamma_{\sigma}p^{\nu}+g^{\nu}_{\sigma}\gamma^{\mu}p_{\rho}-g^{\nu}_{\sigma}\gamma_{\rho}p^{\mu})
\end{aligned}
$$

$$
\begin{aligned}
A_2(\tfrac{3}{2})^{\mu\nu}_{\sigma\rho} = {} & -\frac{1}{2p^2}(g^{\mu}_{\rho}p^{\nu}p_{\sigma}+g^{\nu}_{\sigma}p^{\mu}p_{\rho}-g^{\nu}_{\rho}p^{\mu}p_{\sigma}-g^{\mu}_{\sigma}p^{\nu}p_{\rho}) \\
& -\frac{1}{6p^2}(\gamma^{\nu}\gamma_{\rho}p^{\mu}p_{\sigma}+\gamma^{\mu}\gamma_{\sigma}p^{\nu}p_{\rho}-\gamma^{\mu}\gamma_{\rho}p^{\nu}p_{\sigma}-\gamma^{\nu}\gamma_{\sigma}p^{\mu}p_{\rho})
\end{aligned}
$$

$$
\begin{aligned}
A_1(\tfrac{1}{2})^{\mu\nu}_{\sigma\rho} = {} & -\tfrac{1}{6}(g^{\mu}_{\sigma}g^{\nu}_{\rho}-g^{\mu}_{\rho}g^{\nu}_{\sigma})-\tfrac{1}{6}(g^{\nu}_{\sigma}\gamma^{\mu}\gamma_{\rho}+g^{\mu}_{\rho}\gamma^{\nu}\gamma_{\sigma}-g^{\mu}_{\sigma}\gamma^{\nu}\gamma_{\rho}-g^{\nu}_{\rho}\gamma^{\mu}\gamma_{\sigma}) \\
& -\frac{1}{6p^2}(\gamma^{\nu}\gamma_{\rho}p^{\mu}p_{\sigma}+\gamma^{\mu}\gamma_{\sigma}p^{\nu}p_{\rho}-\gamma^{\mu}\gamma_{\rho}p^{\nu}p_{\sigma}-\gamma^{\nu}\gamma_{\sigma}p^{\mu}p_{\rho}) \\
& -\frac{\gamma\cdot p}{6p^2}(g^{\nu}_{\rho}\gamma_{\sigma}p^{\mu}-g^{\nu}_{\rho}\gamma^{\mu}p_{\sigma}+g^{\mu}_{\sigma}\gamma_{\rho}p^{\nu}-g^{\mu}_{\sigma}\gamma^{\nu}p_{\rho}+g^{\mu}_{\rho}\gamma^{\nu}p_{\sigma}-g^{\mu}_{\rho}\gamma_{\sigma}p^{\nu}+g^{\nu}_{\sigma}\gamma^{\mu}p_{\rho}-g^{\nu}_{\sigma}\gamma_{\rho}p^{\mu})
\end{aligned}
$$

$$
A_2(\tfrac{1}{2})^{\mu\nu}_{\sigma\rho} = \frac{1}{6p^2}(\gamma^{\nu}\gamma_{\rho}p^{\mu}p_{\sigma}+\gamma^{\mu}\gamma_{\sigma}p^{\nu}p_{\rho}-\gamma^{\mu}\gamma_{\rho}p^{\nu}p_{\sigma}-\gamma^{\nu}\gamma_{\sigma}p^{\mu}p_{\rho}).
$$

References

Aurilia A and Umezawa H 1967 *Nuovo Cim.*A **51** 14

—— 1969 *Phys. Rev.* **182** 1682

Johnson K and Sudarsham E C G 1961 *Ann. Phys., NY* **13** 126

Lurié D 1968 *Particles and Fields* (New York: Wiley) pp 45–7

Macfarlane A J and Tait W 1972 *Commun. math. Phys.* **24** 211

Madore J and Tait W 1973 *Commun. math. Phys.* to be published

Minkowski P and Seiler R 1971 *Phys. Rev. D* **4** 359

Schroer B, Seiler R and Swieca J A 1970 *Phys. Rev. D* **2** 2927

Shamaly A and Capri A Z 1972 *Lett. Nuovo Cim.* **3** 467

Stellmacher F K 1938 *Math. Ann.* **115** 740

Velo G and Zwanziger D 1969a *Phys. Rev.* **186** 1337

—— 1969b *Phys. Rev.* **188** 2218